LB1 Funktionale Zusammenhänge: **Nullstellen ganzrationaler Funktionen -** Lösungen

1. $6x + \frac{3}{8} = 0$ → $\underline{x_0 = -\frac{1}{16}}$

2. $\frac{5}{6}x + \frac{1}{2} - \frac{2}{3}x - \frac{7}{4} = 0$ → $\underline{x_0 = \frac{15}{2}}$

3. $-\frac{1}{4}x^2 + x + 3 = 0$ → $\underline{x_{01} = 6}$ $\quad \underline{x_{02} = -2}$

4. $-8x^2 + 37x - 20 = 0$ → $\underline{x_{01} = 4}$ $\quad \underline{x_{02} = \frac{5}{8} = 0{,}625}$

5. $\frac{3}{7}x^3 + \frac{7}{2}x^2 = 0$ → $\underline{x_{01} = 0}$ $\quad \underline{x_{02} = -\frac{49}{6} \approx -8{,}167}$

6. $x^3 + 5x^2 - 4x - 20 = 0$

   $\underline{x_{01} = 2}$ → $(x^3 + 5x^2 - 4x - 20) : (x - 2) = x^2 + 7x + 10$ → $\underline{x_{02} = -2}$ $\quad \underline{x_{03} = -5}$

   $\underline{x_{01} = -2}$ → $(x^3 + 5x^2 - 4x - 20) : (x + 2) = x^2 + 3x - 10$ → $\underline{x_{02} = 2}$ $\quad \underline{x_{03} = -5}$

   $\underline{x_{01} = -5}$ → $(x^3 + 5x^2 - 4x - 20) : (x + 5) = x^2 - 4$ → $\underline{x_{02} = 2}$ $\quad \underline{x_{03} = -2}$

7. $9x^4 - 13x^2 + 4 = 0$ $\;|:9$ → $x^4 - \frac{13}{9}x^2 + \frac{4}{9} = 0$ → Subst.: $x^2 = z$ → $z^2 - \frac{13}{9}z + \frac{4}{9} = 0$

   → $\underline{z_1 = \frac{4}{9}}$ → Rücksubst.: $x^2 = \frac{4}{9}$ → $\underline{x_{01} = \frac{2}{3}}$ $\quad \underline{x_{02} = -\frac{2}{3}}$

   oder $\underline{z_2 = 1}$ → Rücksubst.: $x^2 = 1$ → $\underline{x_{03} = 1}$ $\quad \underline{x_{04} = -1}$

8. $x^4 + 10x^3 + 14x^2 + 35x + 24 = 0$

   A) $\underline{x_{01} = -1}$ → $(x^4 + 10x^3 + 14x^2 + 35x + 24) : (x + 1) = x^3 + 9x^2 + 26x + 24$

   B) $\underline{x_{01} = -2}$ → $(x^4 + 10x^3 + 14x^2 + 35x + 24) : (x + 2) = x^3 + 8x^2 + 19x + 12$

   C) $\underline{x_{01} = -3}$ → $(x^4 + 10x^3 + 14x^2 + 35x + 24) : (x + 3) = x^3 + 7x^2 + 14x + 8$

   D) $\underline{x_{01} = -4}$ → $(x^4 + 10x^3 + 14x^2 + 35x + 24) : (x + 4) = x^3 + 6x^2 + 11x + 6$

   zu A) $\underline{x_{02} = -2}$ → $(x^3 + 9x^2 + 26x + 24) : (x + 2) = x^2 + 7x + 12$ → $\underline{x_{03} = -3}$ $\quad \underline{x_{04} = -4}$

   $\underline{x_{02} = -3}$ → $(x^3 + 9x^2 + 26x + 24) : (x + 3) = x^2 + 6x + 8$ → $\underline{x_{03} = -2}$ $\quad \underline{x_{04} = -4}$

   $\underline{x_{02} = -4}$ → $(x^3 + 9x^2 + 26x + 24) : (x + 4) = x^2 + 5x + 6$ → $\underline{x_{03} = -2}$ $\quad \underline{x_{04} = -3}$

   zu B) $\underline{x_{02} = -1}$ → $(x^3 + 8x^2 + 19x + 12) : (x + 1) = x^2 + 7x + 12$ → $\underline{x_{03} = -3}$ $\quad \underline{x_{04} = -4}$

   $\underline{x_{02} = -3}$ → $(x^3 + 8x^2 + 19x + 12) : (x + 3) = x^2 + 5x + 4$ → $\underline{x_{03} = -1}$ $\quad \underline{x_{04} = -4}$

   $\underline{x_{02} = -4}$ → $(x^3 + 8x^2 + 19x + 12) : (x + 4) = x^2 + 4x + 3$ → $\underline{x_{03} = -1}$ $\quad \underline{x_{04} = -3}$

   zu C) $\underline{x_{02} = -1}$ → $(x^3 + 7x^2 + 14x + 8) : (x + 1) = x^2 + 6x + 8$ → $\underline{x_{03} = -2}$ $\quad \underline{x_{04} = -4}$

   $\underline{x_{02} = -2}$ → $(x^3 + 7x^2 + 14x + 8) : (x + 2) = x^2 + 5x + 4$ → $\underline{x_{03} = -1}$ $\quad \underline{x_{04} = -4}$

   $\underline{x_{02} = -4}$ → $(x^3 + 7x^2 + 14x + 8) : (x + 4) = x^2 + 3x + 2$ → $\underline{x_{03} = -1}$ $\quad \underline{x_{04} = -2}$

   zu D) $\underline{x_{02} = -1}$ → $(x^3 + 6x^2 + 11x + 6) : (x + 1) = x^2 + 5x + 6$ → $\underline{x_{03} = -2}$ $\quad \underline{x_{04} = -3}$

   $\underline{x_{02} = -2}$ → $(x^3 + 6x^2 + 11x + 6) : (x + 2) = x^2 + 4x + 3$ → $\underline{x_{03} = -1}$ $\quad \underline{x_{04} = -3}$

   $\underline{x_{02} = -3}$ → $(x^3 + 6x^2 + 11x + 6) : (x + 3) = x^2 + 3x + 2$ → $\underline{x_{03} = -1}$ $\quad \underline{x_{04} = -2}$

9. $2x^7 + 3x^4 - 2x = 0$ → $2x \cdot (x^6 + \frac{3}{2}x^3 - 1) = 0$ → $\underline{x_{01} = 0}$ oder $x^6 + \frac{3}{2}x^3 - 1 = 0$

   → Subst.: $x^3 = z$ → $z^2 + \frac{3}{2}z - 1 = 0$

   → $\underline{z_1 = \frac{1}{2}}$ → Rücksubst.: $x^3 = \frac{1}{2}$ → $\underline{x_{02} = \sqrt[3]{\frac{1}{2}} \approx 0{,}7937}$

   oder $\underline{z_2 = -2}$ → Rücksubst.: $x^3 = -2$ → $\underline{x_{03} = \sqrt[3]{-2} \approx -1{,}260}$

10. $x^7 - 6x^5 + 11x^3 - 6x = 0$ → $x \cdot (x^6 - 6x^4 + 11x^2 - 6) = 0$ → $\underline{x_{01} = 0}$ oder $x^6 - 6x^4 + 11x^2 - 6 = 0$

    → Subst.: $x^2 = z$ → $z^3 - 6z^2 + 11z - 6 = 0$

    $\underline{z_{01} = 1}$ → $(z^3 - 6z^2 + 11z - 6) : (x - 1) = z^2 - 5z + 6$ → $\underline{z_{02} = 2}$ $\quad \underline{z_{03} = 3}$

    $\underline{z_{01} = 2}$ → $(z^3 - 6z^2 + 11z - 6) : (x - 2) = z^2 - 4z + 3$ → $\underline{z_{02} = 1}$ $\quad \underline{z_{03} = 3}$

    $\underline{z_{01} = 3}$ → $(z^3 - 6z^2 + 11z - 6) : (x - 3) = z^2 - 3z + 2$ → $\underline{z_{02} = 1}$ $\quad \underline{z_{03} = 2}$

    → Rücksubst.: $x^2 = 1$ → $\underline{x_{02} = 1}$ $\quad \underline{x_{03} = -1}$

    $x^2 = 2$ → $\underline{x_{04} = \sqrt{2}}$ $\quad \underline{x_{05} = -\sqrt{2}}$

    $x^2 = 3$ → $\underline{x_{06} = \sqrt{3}}$ $\quad \underline{x_{07} = -\sqrt{3}}$