

LB 2: Grenzwert - Lösungen

1 a) $\lim_{x \rightarrow \infty} \frac{3x-1}{1+5x} = \underline{\underline{\frac{3}{5}}}$

b) $\lim_{x \rightarrow \infty} \frac{4-x^2}{1+2x^2} = \underline{\underline{-\frac{1}{2}}}$

c) $\lim_{x \rightarrow \infty} \left(4 \cdot \frac{1+2x}{2-x} \right) = \underline{\underline{-8}}$

d) $\lim_{x \rightarrow \infty} \frac{2(1+3x^2)}{5x^2-1} = \underline{\underline{\frac{6}{5}}}$

e) $\lim_{x \rightarrow \infty} \frac{2x}{x^2+1} = \underline{\underline{0}}$

f) $\lim_{x \rightarrow \infty} \frac{4x(1+x)}{x^3+1} = \underline{\underline{0}}$

g) $\lim_{x \rightarrow \infty} \frac{a}{3+x^2} = \underline{\underline{0}}$

h) $\lim_{x \rightarrow 0} (2^{-x} + x^2) = \underline{\underline{1}}$

i) $\lim_{x \rightarrow 3} (x^2(1-2^{-x})) = \underline{\underline{\frac{63}{8}}}$

j) $\lim_{x \rightarrow \pi} \frac{\cos x}{1-\sin x} = \underline{\underline{-1}}$

k) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos^2 x}{1-\sin x} = \underline{\underline{2}}$

l) $\lim_{x \rightarrow \pi} \frac{2}{\sin x} = \underline{\underline{-\infty}}$

2 a) $\lim_{x \rightarrow 4} \frac{x^2-16}{x-4} = \lim_{x \rightarrow 4} \frac{(x-4) \cdot (x+4)}{x-4} = \underline{\underline{8}}$

b) $\lim_{x \rightarrow -\frac{1}{2}} \frac{4x^2-1}{2x+1} = \lim_{x \rightarrow -\frac{1}{2}} \frac{(2x+1) \cdot (2x-1)}{2x+1} = \underline{\underline{-2}}$

c) $\lim_{x \rightarrow -1} \frac{4x^2+4x}{x+1} = \lim_{x \rightarrow -1} \frac{4x \cdot (x+1)}{x+1} = \underline{\underline{-4}}$

d) $\lim_{x \rightarrow 4} \frac{2x^2-32}{3x-12} = \lim_{x \rightarrow 4} \frac{(x-4) \cdot (2x+8)}{3 \cdot (x-4)} = \underline{\underline{\frac{16}{3}}}$

e) $\lim_{x \rightarrow 4} \frac{x^2-8x+16}{x-4} = \lim_{x \rightarrow 4} \frac{(x-4)^2}{x-4} = \underline{\underline{0}}$

f) $\lim_{x \rightarrow 1} \frac{1-x}{1-\sqrt{x}} = \lim_{x \rightarrow 1} \frac{(1-\sqrt{x}) \cdot (1+\sqrt{x})}{1-\sqrt{x}} = \underline{\underline{2}}$

3 a) $\lim_{x \rightarrow -2} \left(\frac{x^2-4}{x+2} \right)^2 = \lim_{x \rightarrow -2} (x-2)^2 = \underline{\underline{16}}$

b) $\lim_{x \rightarrow 2} \left(\frac{x-2}{x^2-4} \right)^2 = \lim_{x \rightarrow 2} \left(\frac{1}{x+2} \right)^2 = \underline{\underline{\frac{1}{16}}}$

c) $\lim_{x \rightarrow 5} \left(\frac{x^2-25}{x+5} \right)^2 = \lim_{x \rightarrow 5} (x-5)^2 = \underline{\underline{100}}$

d) $\lim_{x \rightarrow -3} \left(\frac{x^2+x-6}{x+3} \right)^3 = \lim_{x \rightarrow -3} (x-2)^3 = \underline{\underline{-125}}$

e) $\lim_{x \rightarrow 1} \sqrt{\frac{x-1}{x^2-1}} = \lim_{x \rightarrow 1} \sqrt{\frac{1}{x+1}} = \underline{\underline{\frac{1}{2}}}$

f) $\lim_{x \rightarrow 0} \sqrt{1+4 \cdot 2^{-x}} = \underline{\underline{\sqrt{5}}}$

g) $\lim_{x \rightarrow \infty} \left(\frac{2}{1+2x} \right)^2 = \underline{\underline{0}}$

h) $\lim_{x \rightarrow \infty} \left(\frac{x}{1+2^x} \right)^2 = \underline{\underline{\infty}}$

i) $\lim_{x \rightarrow \infty} \frac{1}{\lg x} = \underline{\underline{0}}$

j) $\lim_{x \rightarrow \infty} \ln \frac{2x^2+1}{x^2+1} = \underline{\underline{\ln 2}}$

k) $\lim_{x \rightarrow 0} 2^{\frac{-1}{|x|}} = \lim_{x \rightarrow 0} \frac{1}{2^{\frac{1}{|x|}}} = \underline{\underline{0}}$

l) $\lim_{x \rightarrow 0} \sin^2(\frac{\pi}{4}-x) = \underline{\underline{\frac{1}{2}}}$