

LB 3: Lokale Extrempunkte - Lösungen

1) $f(x) = 2x^2 - 4x + 2$

$\rightarrow f'(x) = 4x - 4 \rightarrow 4x - 4 = 0 \rightarrow x_E = 1$

$\rightarrow f''(x) = 4 \rightarrow f''(1) = 4 > 0 \rightarrow \text{Minimum } f(1) = 0 \rightarrow \underline{T(1; 0)}$

2) $f(x) = -\frac{2}{3}x^2 + \frac{1}{2}x - \frac{1}{3}$

$\rightarrow f'(x) = -\frac{4}{3}x + \frac{1}{2} \rightarrow -\frac{4}{3}x + \frac{1}{2} = 0 \rightarrow x_E = \frac{3}{8} = 0,375$

$\rightarrow f''(x) = -\frac{4}{3} < 0 \rightarrow \text{Maximum } f\left(\frac{3}{8}\right) = \frac{25}{32} = -0,2396 \rightarrow \underline{H(0,375; -0,2396)}$

3) $f(x) = 3x^3 - 6x^2 + 4x - 5$

$\rightarrow f'(x) = 9x^2 - 12x + 4 \rightarrow 9x^2 - 12x + 4 = 0 \rightarrow x_E = \frac{2}{3} \approx 0,667$

$\rightarrow f''(x) = 18x - 12 \rightarrow f''\left(\frac{2}{3}\right) = 0 \rightarrow \underline{\text{kein Extrempunkt}}$

4) $f(x) = -x^3 + 4x^2 + 4x + 4$

$\rightarrow f'(x) = -3x^2 + 8x + 4 \rightarrow -3x^2 + 8x + 4 = 0 \rightarrow x_{E1} \approx -0,4305 \quad x_{E2} \approx 3,0972$

$\rightarrow f''(x) = -6x + 8$

$\rightarrow f''(-0,4305) \approx 10,6 > 0 \rightarrow \text{Minimum } f(-0,4305) \approx 3,1 \rightarrow \underline{T(-0,43; 3,1)}$

$\rightarrow f''(3,0972) \approx -10,6 < 0 \rightarrow \text{Maximum } f(3,0972) \approx 25,05 \rightarrow \underline{H(3,0972; 25,05)}$

5) $f(x) = x^4 - 2x^3 + x^2 - 2x + 1$

$\rightarrow f'(x) = 4x^3 - 6x^2 + 2x - 2 \rightarrow 4x^3 - 6x^2 + 2x - 2 = 0 \rightarrow x_E \approx 1,398$

$\rightarrow f''(x) = 12x^2 - 12x + 2 \rightarrow f''(1,398) \approx 8,7 > 0 \rightarrow \text{Minimum } f(1,398) \approx -1,4864 \rightarrow \underline{T(1,398; -1,486)}$

6) $f(x) = -\frac{1}{3}x^4 + \frac{1}{2}x^3 + \frac{3}{2}x^2 - \frac{2}{3}x + \frac{4}{3}$

$\rightarrow f'(x) = -\frac{4}{3}x^3 + \frac{3}{2}x^2 + 3x - \frac{2}{3} \rightarrow -\frac{4}{3}x^3 + \frac{3}{2}x^2 + 3x - \frac{2}{3} = 0 \rightarrow x_{E1} \approx -1,168 \quad x_{E2} \approx 0,205 \quad x_{E3} \approx 2,088$

$\rightarrow f''(x) = -4x^2 + 3x + 3$

$\rightarrow f''(-1,168) \approx -5,96 < 0 \rightarrow \text{Maximum } f(-1,168) \approx 2,741 \rightarrow \underline{H(-1,168; 2,741)}$

$\rightarrow f''(0,205) \approx 3,45 > 0 \rightarrow \text{Minimum } f(0,205) \approx 1,2634 \rightarrow \underline{T(0,205; 1,263)}$

$\rightarrow f''(2,088) \approx -8,17 < 0 \rightarrow \text{Maximum } f(2,088) \approx 4,6967 \rightarrow \underline{H(2,088; 4,6967)}$