1) $f_k(x) = (k-1)x^3 - 2x^2 + \frac{1}{k-1}x$, $k \in R, k > 1$

a) die Nullstellen:

$(k-1)x^3 - 2x^2 + \frac{1}{k-1}x = 0 \rightarrow x \cdot \left((k-1)x^2 - 2x + \frac{1}{k-1}\right) = 0 \rightarrow \underline{\underline{x_{01} = 0}}$ und $(k-1)x^2 - 2x + \frac{1}{k-1} = 0 \quad |:(k-1)$

$\rightarrow x^2 - \frac{2}{k-1}x + \frac{1}{(k-1)^2} = 0$

$\rightarrow x_{02,3} = \frac{1}{k-1} \pm \sqrt{\frac{1}{(k-1)^2} - \frac{1}{(k-1)^2}} = \underline{\underline{\frac{1}{k-1}}}$

b) lokale Extrempunkte:

$f_k'(x) = 3(k-1)x^2 - 4x + \frac{1}{k-1} \rightarrow 3(k-1)x^2 - 4x + \frac{1}{k-1} = 0 \quad |:(3(k-1)) \rightarrow x^2 - \frac{4}{3(k-1)}x + \frac{1}{3(k-1)^2} = 0$

$\rightarrow x_{E1,2} = \frac{2}{3(k-1)} \pm \sqrt{\frac{4}{9(k-1)^2} - \frac{1}{3(k-1)^2}} = \frac{2}{3(k-1)} \pm \sqrt{\frac{4}{9(k-1)^2} - \frac{3}{9(k-1)^2}} \rightarrow \underline{x_{E1} = \frac{1}{k-1}} \quad \underline{x_{E2} = \frac{1}{3(k-1)}}$

$f_k''(x) = 6(k-1)x - 4$

$\rightarrow f_k''\left(\frac{1}{k-1}\right) = 2 > 0 \rightarrow$ Minimum $\underline{\underline{T\left(\frac{1}{k-1}; 0\right)}}$

$\rightarrow f_k''\left(\frac{1}{3(k-1)}\right) = -2 < 0 \rightarrow$ Maximum und $f_k\left(\frac{1}{3(k-1)}\right) = \frac{4}{27(k-1)^2} \rightarrow \underline{\underline{H\left(\frac{1}{3(k-1)}; \frac{4}{27(k-1)^2}\right)}}$

c) Wendepunkte:

$f_k''(x) = 6(k-1)x - 4 \rightarrow 6(k-1)x - 4 = 0 \rightarrow \underline{x_W = \frac{2}{3(k-1)}}$

$f_k'''(x) = 6(k-1) \neq 0$ und $f_k\left(\frac{2}{3(k-1)}\right) = \frac{2}{27(k-1)^2} \rightarrow \underline{W\left(\frac{2}{3(k-1)}; \frac{2}{27(k-1)^2}\right)}$

2) $f_t(x) = x^3 - (t^2 + 2t)x^2$, $t \in R$

a) die Nullstellen:

$x^3 - (t^2 + 2t)x^2 = 0 \rightarrow x^2(x - (t^2 + 2t)) = 0 \rightarrow \underline{\underline{x_{01} = 0}}$ und $\underline{\underline{x_{02} = t^2 + 2t}}$

b) lokale Extrempunkte:

$f_t'(x) = 3x^2 - 2x(t^2 + 2t) \rightarrow 3x^2 - 2x(t^2 + 2t) = 0 \rightarrow x(3x - 2(t^2 + 2t)) = 0 \rightarrow \underline{x_{E1} = 0} \quad \underline{x_{E2} = \frac{2}{3}(t^2 + 2t)}$

$f_t''(x) = 6x - 2(t^2 + 2t) \rightarrow f_t''(0) = -2t(t+2)$
$\rightarrow$ für $t < -2$ oder $t > 0 : f_t''(0) < 0 \rightarrow$ Maximum
$\rightarrow$ für $-2 < t < 0 : f_t''(0) > 0 \rightarrow$ Minimum
$\rightarrow$ für $t = -2$ oder $t = 0 : f_t''(0) = 0 \rightarrow$ keine Extrempkt.

$\rightarrow f_t''\left(\frac{2}{3}(t^2 + 2t)\right) = 2t(t+2)$
$\rightarrow$ für $t < -2$ oder $t > 0 : f_t''(0) > 0 \rightarrow$ Minimum
$\rightarrow$ für $-2 < t < 0 : f_t''(0) < 0 \rightarrow$ Maximum

$f_t(0) = 0$ und $f_t\left(\frac{2}{3}(t^2 + 2t)\right) = -\frac{4}{27}(t^2 + 2t)^3$

$\rightarrow$ für $t < -2$ oder $t > 0$: $\underline{\underline{H(0; 0)}}$ und $\underline{\underline{T\left(\frac{2}{3}(t^2 + 2t); -\frac{4}{27}(t^2 + 2t)^3\right)}}$

für $-2 < t < 0$: $\underline{\underline{T(0; 0)}}$ und $\underline{\underline{H\left(\frac{2}{3}(t^2 + 2t); -\frac{4}{27}(t^2 + 2t)^3\right)}}$

für $t = -2$ oder $t = 0$: $\rightarrow$ keine Extrempkt.

3) $f_{a,b}(x) = ax^4 + bx^3 - \frac{b^2}{2a} \cdot x^2,\ a,b \in N^*$

a) Nullstellen:

$$ax^4 + bx^3 - \frac{b^2}{2a} \cdot x^2 = 0 \rightarrow x^2\left(ax^2 + bx - \frac{b^2}{2a}\right) = 0 \rightarrow \underline{x_{01} = 0} \text{ und } ax^2 + bx - \frac{b^2}{2a} = 0 \rightarrow x^2 + \frac{b}{a}x - \frac{b^2}{2a^2} = 0$$

$$\rightarrow x_{02,3} = -\frac{b}{2a^2} \pm \sqrt{\frac{b^2}{4a^2} + \frac{2b^2}{4a^2}} = -\frac{b}{2a} \pm \frac{b}{2a}\sqrt{3} = \underline{\underline{-\frac{b}{2a}\left(1 \pm \sqrt{3}\right)}}$$

b) lokale Extrempunkte:

$$f'_{a,b}(x) = 4ax^3 + 3bx^2 - \frac{b^2}{a}x \rightarrow 4ax^3 + 3bx^2 - \frac{b^2}{a}x = 0 \rightarrow x\left(4ax^2 + 3bx - \frac{b^2}{a}\right) = 0$$

$$\rightarrow \underline{x_{E1} = 0} \text{ und } 4ax^2 + 3bx - \frac{b^2}{a} = 0 \rightarrow x^2 + \frac{3b}{4a}x - \frac{b^2}{4a^2} = 0 \rightarrow x_{E2,3} = -\frac{3b}{8a} \pm \sqrt{\frac{9b^2}{64a^2} + \frac{16b^2}{64a^2}}$$

$$\rightarrow \underline{x_{E2} = \frac{b}{4a}} \quad \underline{x_{E3} = -\frac{b}{a}}$$

$$f''_{a,b}(x) = 12ax^2 + 6bx - \frac{b^2}{a}$$

$$f''_{a,b}(0) = -\frac{b^2}{a} < 0 \rightarrow \text{Maximum und } f_{a,b}(0) = 0 \rightarrow \underline{\underline{H(0;\,0)}}$$

$$f''_{a,b}\left(\frac{b}{4a}\right) = \frac{5b^2}{4a} > 0 \rightarrow \text{Minimum und } f_{a,b}\left(\frac{b}{4a}\right) = -\frac{3b^4}{256a^3} \rightarrow \underline{\underline{T\left(\frac{b}{4a};\, -\frac{3b^4}{256a^3}\right)}}$$

$$f''_{a,b}\left(-\frac{b}{a}\right) = \frac{5b^2}{a} > 0 \rightarrow \text{Minimum und } f_{a,b}\left(-\frac{b}{a}\right) = -\frac{b^4}{2a^3} \rightarrow \underline{\underline{T\left(-\frac{b}{a};\, -\frac{b^4}{2a^3}\right)}}$$