Geg.: Punkte A(1;2;3), B(–2;–3;–3) , C(0;3;2), D(2;3;-2)

1. a) $\overrightarrow{AB}=\begin{pmatrix}-3\\-5\\-6\end{pmatrix}$ b) $\overrightarrow{AC}=\begin{pmatrix}-1\\1\\-1\end{pmatrix}$ c) $\overrightarrow{AD}=\begin{pmatrix}1\\1\\-5\end{pmatrix}$ d) $\overrightarrow{BC}=\begin{pmatrix}2\\6\\5\end{pmatrix}$ e) $\overrightarrow{BD}=\begin{pmatrix}4\\6\\1\end{pmatrix}$ f) $\overrightarrow{CD}=\begin{pmatrix}2\\0\\-4\end{pmatrix}$

2. a) $-3\overrightarrow{AB}+2\overrightarrow{AC}=\begin{pmatrix}7\\17\\16\end{pmatrix}$ b) $2\overrightarrow{AD}-\frac{1}{2}\overrightarrow{BC}=\begin{pmatrix}1\\-1\\-12{,}5\end{pmatrix}$ c) $-\frac{2}{5}\overrightarrow{BD}-\frac{1}{2}\overrightarrow{CD}=\begin{pmatrix}-2{,}6\\-2{,}4\\1{,}6\end{pmatrix}$

3. a) $\left|-3\overrightarrow{AB}\right|+\left|2\overrightarrow{AC}\right|\approx 25{,}1+3{,}46\approx 28{,}6$ b) $2\left|\overrightarrow{AD}\right|-\frac{1}{2}\left|\overrightarrow{BC}\right|\approx 10{,}39-4{,}03\approx 6{,}36$ c) $\left|-\frac{2}{5}\overrightarrow{BD}-\frac{1}{2}\overrightarrow{CD}\right|\approx 3{,}88$

4. a) $\overrightarrow{AB}\bullet\overrightarrow{AC}=4$ b) $\left(\frac{1}{3}\overrightarrow{AD}\right)\bullet\left(2\overrightarrow{BC}\right)=-11\frac{1}{3}$ c) $\left(-\frac{3}{5}\overrightarrow{BD}\right)\bullet\left(-\frac{4}{3}\overrightarrow{CD}\right)=3{,}2$

5. a) $\overrightarrow{AB}\,x\,\overrightarrow{AC}=\begin{pmatrix}11\\3\\-8\end{pmatrix}$ b) $\left(\frac{1}{3}\overrightarrow{AD}\right)x\left(2\overrightarrow{BC}\right)=\begin{pmatrix}\frac{70}{3}\\-10\\\frac{8}{3}\end{pmatrix}$ c) $\left(-\frac{3}{5}\overrightarrow{BD}\right)x\left(-\frac{4}{3}\overrightarrow{CD}\right)=\frac{4}{5}\begin{pmatrix}-24\\18\\-12\end{pmatrix}=\begin{pmatrix}-19{,}2\\14{,}4\\-9{,}6\end{pmatrix}$

6. a) $\left|\overrightarrow{AB}\,x\,\overrightarrow{AC}\right|=\sqrt{194}\approx 13{,}9$ b) $\left|\left(\frac{1}{3}\overrightarrow{AD}\right)x\left(2\overrightarrow{BC}\right)\right|\approx 25{,}5$ c) $\left|\left(-\frac{3}{5}\overrightarrow{BD}\right)x\left(-\frac{4}{3}\overrightarrow{CD}\right)\right|\approx 25{,}8$

7. a) $\left|\overrightarrow{AB}\right|=\left|\begin{pmatrix}-3\\-5\\-6\end{pmatrix}\right|=\sqrt{70}\approx 8{,}37\text{LE}$ $\left|\overrightarrow{AC}\right|=\left|\begin{pmatrix}-1\\1\\-1\end{pmatrix}\right|=\sqrt{3}\approx 1{,}73\text{LE}$ $\left|\overrightarrow{BC}\right|=\left|\begin{pmatrix}2\\6\\5\end{pmatrix}\right|=\sqrt{65}\approx 8{,}06\text{LE}$

   → $u_{ABC}\approx \underline{\underline{18{,}2\text{LE}}}$

   b) $\left|\overrightarrow{AB}\right|=\left|\begin{pmatrix}-3\\-5\\-6\end{pmatrix}\right|=\sqrt{70}\approx 8{,}37\text{LE}$ $\left|\overrightarrow{AD}\right|=\left|\begin{pmatrix}1\\1\\-5\end{pmatrix}\right|=\sqrt{27}\approx 5{,}20\text{LE}$ $\left|\overrightarrow{BD}\right|=\left|\begin{pmatrix}4\\6\\1\end{pmatrix}\right|=\sqrt{53}\approx 7{,}28\text{LE}$

   → $u_{ABD}\approx \underline{\underline{20{,}8\text{LE}}}$

   c) $\left|\overrightarrow{BC}\right|=\left|\begin{pmatrix}2\\6\\5\end{pmatrix}\right|=\sqrt{65}\approx 8{,}06\text{LE}$ $\left|\overrightarrow{BD}\right|=\left|\begin{pmatrix}4\\6\\1\end{pmatrix}\right|=\sqrt{53}\approx 7{,}28\text{LE}$

   $\left|\overrightarrow{CD}\right|=\left|\begin{pmatrix}2\\0\\-4\end{pmatrix}\right|=\sqrt{20}\approx 4{,}47\text{LE}$

   → $u_{ABC}\approx \underline{\underline{19{,}8\text{LE}}}$

8. a) $\underline{S_{ABC}\left(-\frac{1}{3};\frac{2}{3};\frac{2}{3}\right)}$ b) $\underline{S_{ABD}\left(\frac{1}{3};\frac{2}{3};-\frac{2}{3}\right)}$) $\underline{S_{BCD}(0;1;-1)}$

9. a) $\cos\alpha = \dfrac{\begin{pmatrix}-3\\-5\\-6\end{pmatrix} \circ \begin{pmatrix}-1\\1\\-1\end{pmatrix}}{\sqrt{70\cdot 3}} = \dfrac{4}{\sqrt{210}} \approx 0{,}2760 \rightarrow \underline{\underline{\alpha \approx 74{,}0°}}$

b) $\cos\beta = \dfrac{\begin{pmatrix}2\\6\\5\end{pmatrix} \circ \begin{pmatrix}4\\6\\1\end{pmatrix}}{\sqrt{65\cdot 53}} = \dfrac{49}{\sqrt{65\cdot 53}} \approx 0{,}8348 \rightarrow \underline{\beta \approx 33{,}4°}$

a) $\cos\gamma = \dfrac{\begin{pmatrix}1\\-1\\1\end{pmatrix} \circ \begin{pmatrix}2\\0\\-4\end{pmatrix}}{\sqrt{3\cdot 20}} = \dfrac{-2}{\sqrt{60}} \approx -0{,}2580 \rightarrow \underline{\gamma \approx 105{,}0°}$

10. a) $A = \left|\begin{pmatrix}-3\\-5\\-6\end{pmatrix} x \begin{pmatrix}-1\\1\\-1\end{pmatrix}\right| = \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\-3 & -5 & -6\\-1 & 1 & -1\end{vmatrix} = \left|\begin{pmatrix}11\\3\\-8\end{pmatrix}\right| = \sqrt{121+9+64} = \sqrt{194} \approx \underline{13{,}9\text{FE}}$

b) $A = \left|\begin{pmatrix}-1\\1\\-1\end{pmatrix} x \begin{pmatrix}1\\1\\-5\end{pmatrix}\right| = \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\-1 & 1 & -1\\1 & 1 & -5\end{vmatrix} = \left|\begin{pmatrix}-4\\-6\\-2\end{pmatrix}\right| = \sqrt{16+36+4} = \sqrt{56} \approx \underline{\underline{7{,}5\text{FE}}}$

c) $A = \dfrac{1}{2}\left|\begin{pmatrix}-3\\-5\\-6\end{pmatrix} x \begin{pmatrix}4\\6\\1\end{pmatrix}\right| = \dfrac{1}{2}\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\-3 & -5 & -6\\4 & 6 & 1\end{vmatrix} = \dfrac{1}{2}\left|\begin{pmatrix}31\\-21\\2\end{pmatrix}\right| = \dfrac{1}{2}\sqrt{31^2+21^2+4} \approx \underline{18{,}75\text{FE}}$

11. a) $V = \left|\begin{pmatrix}11\\3\\-8\end{pmatrix} \circ \begin{pmatrix}1\\1\\-5\end{pmatrix}\right| = |54| = \underline{54\,\text{VE}}$

b) $V = \dfrac{1}{3}\left|\left(\begin{pmatrix}-3\\-5\\-6\end{pmatrix} x \begin{pmatrix}1\\1\\-5\end{pmatrix}\right) \circ \begin{pmatrix}-1\\1\\-1\end{pmatrix}\right| = \dfrac{1}{3}\left|\begin{pmatrix}31\\-21\\2\end{pmatrix} \circ \begin{pmatrix}-1\\1\\-1\end{pmatrix}\right| = \dfrac{1}{3}|-31-21-2| = \dfrac{1}{3}|-54| \approx \underline{18\,\text{VE}}$

c) $V = \dfrac{1}{6}\left|\left(\begin{pmatrix}2\\6\\5\end{pmatrix} x \begin{pmatrix}4\\6\\1\end{pmatrix}\right) \circ \begin{pmatrix}3\\5\\6\end{pmatrix}\right| = \dfrac{1}{6}\left|\begin{pmatrix}-24\\18\\-12\end{pmatrix} \circ \begin{pmatrix}3\\5\\6\end{pmatrix}\right| = \dfrac{1}{6}|-72+90-72| = \dfrac{1}{6}|-54| \approx \underline{9\,\text{VE}}$